

Spectroscopy of pentaquark states

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Abstract

We construct a complete classification of $qqqq\bar{q}$ pentaquark states in terms of the spin-flavour $SU(6)$ representations. We find that only some definite $SU(3)$ representations are allowed, that is singlets, octets, decuplets, anti-decuplets, 27-plets and 35-plets. The latter three contain exotic states, which cannot be constructed from three quarks only. This complete classification is general and model independent and is useful both for model builders and experimentalists. The mass spectrum is obtained from a Gürsey-Radicati type mass formula, whose coefficients have been determined previously by a study of qqq baryons. The ground state pentaquark, which is identified with the recently observed $\Theta^+(1540)$ state, is predicted to be an isosinglet anti-decuplet state. Its parity depends on the interplay between the spin-flavour and orbital contributions to the mass operator.

1 Introduction

Recently, a baryon with positive strangeness $S = +1$ has been identified by several experimental groups [1, 2, 3, 4, 5, 6, 7]. A second exotic baryon with charge $Q = -2$ has also been observed [8]. These states are exotic in the sense that they cannot be built up from three quarks only as is the case for standard baryons. A state with $S = +1$ or $Q = -2$ requires at least a pentaquark configuration of the type $qqqq\bar{q}$.

The possibility and the interest for $S = +1$ baryons (or Z baryons) has been recorded for many years by the PDG up to 1986, but subsequently it was dropped because of lack of clear evidence for their existence. However, theoretical interest in exotic baryons has continued both for heavy (see [9]) and light quarks (see [10, 11, 12, 13]).

The experimental interest in pentaquarks was triggered by the work of Diakonov et al. [14], who predicted an exotic $S = +1$ baryon with a definite mass and a small width, thus providing an invaluable guide for experimentalists. Such a state, the now famous Θ^+ , is the isoscalar member of a flavour anti-decuplet, whose relative energies are evaluated by means of a $SU_f(3)$ violating interaction based on the Skyrme model. The energy scale is fixed identifying the nucleon-like state with $S = 0$ of the anti-decuplet with the well-known $N(1710)$ resonance. In this way the obtained value of the spin and parity of the Θ^+ is $\frac{1}{2}^+$. However, from the experimental point of view, the known properties of Θ^+ are: the mass (in remarkably coincidence with the prediction of [14]), the width (smaller than the one of other N^* resonances of comparable mass, in qualitative agreement with the prediction [14]), the strangeness ($S = +1$) and the charge ($Q = +1$). Moreover, it seems to be an isosinglet [3]. In this way it can be safely identified with the isoscalar state of the anti-decuplet. On the contrary, the spin and the parity still have to be determined.

The discovery of the pentaquark has produced a strongly increased theoretical interest, giving rise to a long series of papers which address various aspects of pentaquarks. Besides the Skyrme model [13, 14, 15, 16, 17], there are many studies based on the Constituent Quark Model (CQM) [18, 19, 20, 21, 22, 23, 24, 25], the diquark-diquark- \bar{q} approach [26], QCD sum rules [27], large N_c QCD [28], lattice QCD [29], and many others [30]. In many cases the models assume or predict a definite parity for the Θ^+ , which in most cases is positive [14, 15, 18, 21, 23, 26]. However, recent work on QCD-sum rules [27] and lattice QCD [29] implies a negative parity.

In this article, we study the classification scheme of pentaquark states from symmetry principles, leading to a complete basis for the $qqqq\bar{q}$ states in terms of the spin-flavour $SU_{\text{sf}}(6)$ multiplets. Next we calculate the energies of exotic pentaquark states using a Gürsey-Radicati type mass formula, discuss some general features of the pentaquark spectrum, and finally address the properties of the ground state pentaquark state.

2 The classification of pentaquark states

As for all multiquark systems, the pentaquark wave function contains contributions connected to the spatial degrees of freedom and the internal degrees of freedom of colour, flavour and spin. In order to classify the corresponding states, we shall make use as much as possible of symmetry principles without, for the moment, introducing any explicit dynamical model. In the construction of the classification scheme we are guided by two conditions: the pentaquark wave function should be a colour singlet as all physical states, and should be antisymmetric under any permutation of the four quarks.

We shall make use of the Young tableau technique to construct the allowed $SU_{\text{sf}}(6)$ representations for the pentaquark $q^4\bar{q}$ system, denoting with a box the fundamental representation of $SU(n)$, with $n = 2, 3, 6$ for the spin, flavour (or colour), and spin-flavour degrees of freedom, respectively. The quark transforms as the fundamental representation [1] under $SU(n)$, whereas the antiquark transforms as the conjugate representation $[1^{n-1}]$ under $SU(n)$. The spin-flavour classification for the quark and antiquark are given by

$$\begin{array}{rcccl}
& SU_{\text{sf}}(6) & \supset & SU_{\text{f}}(3) & \otimes & SU_{\text{s}}(2) \\
\text{quark} & [1] & \supset & [1] & \otimes & [1] \\
& \square & \supset & \square & \otimes & \square \\
\text{antiquark} & [11111] & \supset & [11] & \otimes & [1] \\
& \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \square \\ \hline \square \\ \hline \end{array} & \supset & \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} & \otimes & \square
\end{array} \tag{1}$$

on the right hand we have used inner products of single quark states. The spin-flavour states of multiquark systems can be obtained by taking the outer product of the representations of the quarks and/or antiquarks.

2.1 The q^3 system

In order to establish the notation, we start by considering the well-known example of qqq baryons. The allowed $SU_{\text{sf}}(6)$ states are obtained by means of the product

$$\square \otimes \square \otimes \square = \square\square\square \oplus 2 \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} \oplus \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \square \\ \hline \end{array} \quad (2)$$

In the following, we adopt for the representations the notation $[f]_d = [f_1, \dots, f_n]_d$, where f_i denotes the number of boxes in the i -th row of the Young tableau, and d is the dimension of the representation. In this way, the above product is written as

$$[1]_6 \otimes [1]_6 \otimes [1]_6 = [3]_{56} \oplus 2[21]_{70} \oplus [111]_{20} . \quad (3)$$

In Table 1 we summarize the results for the allowed spin-flavour, flavour (colour) and spin states of q^3 baryons. The spin states are given by the representations $[f_1 f_2] = [30]$ and $[21]$ or, equivalently, by their spin $s = (f_1 - f_2)/2 = 3/2$ and $1/2$, respectively. On the left-hand side we show the labels of the point group D_3 which is isomorphic to the permutation group of three identical objects S_3 . A complete classification of three quark states involves the analysis of the flavour and spin content of each spin-flavour representation, i.e. the decomposition of representations of $SU_{\text{sf}}(6)$ into those of $SU_{\text{f}}(3) \otimes SU_{\text{s}}(2)$ (see also Table 2)

$$\begin{aligned} [3]_{56} &= ([21]_8 \otimes [21]_2) \oplus ([3]_{10} \otimes [3]_4) , \\ [21]_{70} &= ([21]_8 \otimes [21]_2) \oplus ([21]_8 \otimes [3]_4) \oplus ([3]_{10} \otimes [21]_2) \oplus ([111]_1 \otimes [21]_2) , \\ [111]_{20} &= ([21]_8 \otimes [21]_2) \oplus ([111]_1 \otimes [3]_4) , \end{aligned} \quad (4)$$

or in the usual notation

$$\begin{aligned} [56] &= {}^2 8 \oplus {}^4 10 , \\ [70] &= {}^2 8 \oplus {}^4 8 \oplus {}^2 10 \oplus {}^2 1 , \\ [20] &= {}^2 8 \oplus {}^4 1 . \end{aligned} \quad (5)$$

2.2 The q^4 system

To study the structure of pentaquark $q^4 \bar{q}$ states, it is convenient to first construct the $qqqq$ states which should satisfy Pauli statistics, and then to add the \bar{q} antiquark.

The allowed $SU_{\text{sf}}(6)$ spin-flavour states of the q^4 system follow from the product of the q^3 configurations of Eq. (3) and a single quark

$$\begin{aligned} [3]_{56} \otimes [1]_6 &= [4]_{126} \oplus [31]_{210} , \\ [21]_{70} \otimes [1]_6 &= [31]_{210} \oplus [22]_{105} \oplus [211]_{105} , \\ [111]_{20} \otimes [1]_6 &= [211]_{105} \oplus [1111]_{15} . \end{aligned} \quad (6)$$

As a result we obtain for the q^4 spin-flavour states

$$[1]_6 \otimes [1]_6 \otimes [1]_6 \otimes [1]_6 = [4]_{126} \oplus 3[31]_{210} \oplus 2[22]_{105} \oplus 3[211]_{105} \oplus [1111]_{15} . \quad (7)$$

In Table 3 we summarize the results for the allowed spin-flavour, flavour (colour) and spin states of a system of four identical quarks. The permutation symmetry is characterized by the S_4 Young tableaux $[4]$, $[31]$, $[22]$, $[211]$ and $[1111]$ or, equivalently, by the irreducible representations of the tetrahedral group \mathcal{T}_d (which is isomorphic to S_4) as A_1 , F_2 , E , F_1 and A_2 , respectively. The flavour and spin content of the various q^4 configurations of Eq. (7) is presented in Table 4. The \mathcal{T}_d labels denote the permutation symmetry of the four-quark system, and the D_3 labels that of the three-quark subsystem.

2.3 The $q^4\bar{q}$ system

The pentaquark configurations are now obtained by considering the product of the q^4 states of Eq. (7) and the antiquark state of Eq. (1). The allowed $SU_{\text{sf}}(6)$ states are

$$\begin{aligned}
[4]_{126} \otimes [11111]_6 &= [51111]_{700} \oplus [411111]_{56} , \\
[31]_{210} \otimes [11111]_6 &= [42111]_{1134} \oplus [411111]_{56} \oplus [321111]_{70} , \\
[22]_{105} \otimes [11111]_6 &= [33111]_{560} \oplus [321111]_{70} , \\
[211]_{105} \otimes [11111]_6 &= [32211]_{540} \oplus [321111]_{70} \oplus [222111]_{20} , \\
[1111]_{15} \otimes [11111]_6 &= [22221]_{70} \oplus [222111]_{20} ,
\end{aligned} \tag{8}$$

As a result, we obtain for the $q^4\bar{q}$ spin-flavour states

$$\begin{aligned}
[1]_6 \otimes [1]_6 \otimes [1]_6 \otimes [1]_6 \otimes [11111]_6 &= [51111]_{700} \oplus 4[411111]_{56} \oplus 3[42111]_{1134} \\
&\oplus 8[321111]_{70} \oplus 2[33111]_{560} \oplus 3[32211]_{540} \\
&\oplus 4[222111]_{20} \oplus [22221]_{70} .
\end{aligned} \tag{9}$$

In a similar way, one can construct the allowed flavour multiplets as

$$\begin{aligned}
[1]_3 \otimes [1]_3 \otimes [1]_3 \otimes [1]_3 \otimes [11]_3 &= [51]_{35} \oplus 3[42]_{27} \oplus 2[33]_{\overline{10}} \\
&\oplus 4[411]_{10} \oplus 8[321]_8 \oplus 3[222]_1 .
\end{aligned} \tag{10}$$

The allowed spin states are obtained from

$$[1]_2 \otimes [1]_2 \otimes [1]_2 \otimes [1]_2 \otimes [1]_2 = [5]_6 \oplus 4[41]_4 \oplus 5[32]_2 , \tag{11}$$

where the configurations [5], [41] and [32] correspond to the spin values $s = 5/2, 3/2$ and $1/2$, respectively. In Table 5, we summarize the results for the allowed spin, flavour and spin-flavour states for $q^4\bar{q}$ pentaquarks. The \mathcal{T}_d labels in the last column denote the permutation symmetry of the four-quark subsystem.

The full decomposition of the spin-flavour states of Eq. (9) into the spin and flavour states of Eqs. (11) and (10) is presented in Table 6. The results are in agreement with the reduction of the colour-spin $SU_{\text{cs}}(6)$ algebra of [10]. The spin and flavour content of the $SU_{\text{sf}}(6)$ representations $[411111]_{56}$, $[321111]_{70}$ and $[222111]_{20}$ is the same as that of the representations $[3]_{56}$, $[21]_{70}$ and $[111]_{20}$ for the three-quark system in Eqs. (4) and (5). This means that the states belonging to these representations have the same quantum numbers as the qqq system and hence are difficult to distinguish from the commonly known baryon resonances. Therefore, exotic states, that is pentaquarks having quantum numbers not obtainable with three-quark configurations, are to be looked for in the remaining five $SU_{\text{sf}}(6)$ representations of Eq. (9): $[51111]_{700}$, $[42111]_{1134}$, $[33111]_{560}$, $[32211]_{540}$ and $[22221]_{70}$. Their decomposition into spin and flavour states can be found in Table 6. In the notation of Eq. (5), we can write

$$\begin{aligned}
[700] &= {}^28 \oplus {}^48 \oplus {}^210 \oplus {}^410 \oplus {}^610 \\
&\quad \oplus {}^2\overline{10} \oplus {}^227 \oplus {}^427 \oplus {}^435 \oplus {}^635 , \\
[1134] &= {}^21 \oplus {}^41 \oplus 3({}^28) \oplus 3({}^48) \oplus {}^68 \oplus 2({}^210) \oplus 2({}^410) \oplus {}^610 \\
&\quad \oplus {}^2\overline{10} \oplus {}^4\overline{10} \oplus 2({}^227) \oplus 2({}^427) \oplus {}^627 \oplus {}^235 \oplus {}^435 . \\
[560] &= {}^41 \oplus 2({}^28) \oplus 2({}^48) \oplus {}^68 \oplus {}^210 \oplus {}^410 \\
&\quad \oplus {}^2\overline{10} \oplus {}^4\overline{10} \oplus {}^6\overline{10} \oplus {}^227 \oplus {}^427 \oplus {}^235 , \\
[540] &= {}^21 \oplus {}^41 \oplus {}^61 \oplus 3({}^28) \oplus 3({}^48) \oplus {}^68 \oplus {}^210 \oplus {}^410 \\
&\quad \oplus {}^2\overline{10} \oplus {}^4\overline{10} \oplus 2({}^227) \oplus {}^427 , \\
[\overline{70}] &= {}^21 \oplus {}^28 \oplus {}^48 \oplus {}^2\overline{10} ,
\end{aligned} \tag{12}$$

It is difficult to distinguish the pentaquark flavour singlets, octets and decuplets from the standard three-quark states. The $SU_f(3)$ representations $\overline{10}$, 27 and 35 (see Figs. 1–3) contain exotic states which cannot be obtained from three-quark configurations only. These states are more easily identified experimentally because of the uniqueness of their quantum numbers. In Table 7 we present a complete list of exotic pentaquark states. For each isospin multiplet we have identified the states whose combination of hypercharge Y and charge Q cannot be obtained with three-quark configurations. In Figs. 1–3 the exotic states are indicated by \bullet .

So far, we have discussed the spin-flavour part of the pentaquark wave function with S_4 (or \mathcal{T}_d symmetry). The spin-flavour part has to be combined with the colour part and the orbital (or radial) part in such a way that the total pentaquark wave function is a $[222]_1$ colour-singlet state, and that the four quarks obey the Pauli principle, i.e. are antisymmetric under any permutation of the four quarks. Since the colour part of the pentaquark wave function is a $[222]_1$ singlet and that of the antiquark a $[11]_3$ anti-triplet, the colour wave function of the four-quark configuration is a $[211]_3$ triplet with F_1 symmetry under \mathcal{T}_d . The total q^4 wave function is antisymmetric (A_2), hence the orbital-spin-flavour part is a $[31]$ state with F_2 symmetry which is obtained from the colour part by interchanging rows and columns

$$\begin{array}{lll} \psi_c(q^4) & [211] & F_1 \\ \psi_{\text{osf}}(q^4) & [31] & F_2 \end{array} \quad (13)$$

Next we discuss the symmetry properties of the orbital part of the pentaquark wave function. If the four quarks are in a spatially symmetric S -wave ground state with A_1 symmetry, the only allowed $SU_{\text{sf}}(6)$ representation is $[31]$ with F_2 symmetry. According to Table 6, the only pentaquark configuration with F_2 symmetry that contains exotic states is $[42111]_{1134}$. On the other hand, if the four quarks are in a P -wave state with F_2 symmetry, there are several allowed $SU_{\text{sf}}(6)$ representations: $[4]$, $[31]$, $[22]$ and $[211]$ with A_1 , F_2 , E and F_1 symmetry, respectively. The corresponding pentaquark configurations that contain exotic states are $[51111]_{700}$, $[42111]_{1134}$, $[33111]_{560}$ and $[32211]_{540}$, respectively. In Table 8 we present for each symmetry type of the orbital wave function, the corresponding symmetry of the spin-flavour wave function, as well as the associated pentaquark configurations that contain exotic states. The explicit construction of the S_4 invariant orbital-spin-flavour pentaquark wave functions will be presented in a separate publication [32]. The methods are analogous to those used for the S_3 invariant qqq baryon wave functions (see e.g. [33, 34, 35, 36]).

We would like to stress the general validity of these results. The classification scheme derived in this section is complete, and is based only on the fact that quarks (and antiquarks) have orbital, colour, spin and flavour degrees of freedom. The precise ordering of the pentaquark states in the mass spectrum depends on the choice of a specific dynamical model (Skyrme, CQM, Goldstone Boson Exchange, instanton, hypercentral, stringlike, ...). In the case of the Skyrmion model many states are suppressed because of a strict correlation between spin and isospin.

3 The pentaquark spectrum

In order to study the general structure of the spectrum of exotic pentaquarks, we consider a simple schematic model in which the mass operator is given by

$$M = M_0 + M_{\text{orb}} + M_{\text{sf}} . \quad (14)$$

M_0 is a constant. M_{orb} describes the contribution to the pentaquark mass due to the space degrees of freedom of the constituent quarks. The last term M_{sf} contains the spin-flavour dependence and is assumed to have a generalized Gürsey-Radicati form

$$\begin{aligned} M_{\text{sf}} = & -A C_{2SU_{\text{sf}}(6)} + B C_{2SU_f(3)} + C s(s+1) \\ & + D Y + E [I(I+1) - \frac{1}{4} Y^2] . \end{aligned} \quad (15)$$

The first two terms represent the quadratic Casimir operators of the $SU_{sf}(6)$ spin-flavour and the $SU_f(3)$ flavour groups, and s , Y and I denote the spin, hypercharge and isospin, respectively. For the definition of the Casimir operators in Eq. (15), we have followed the same convention as in [19]. The eigenvalues of the Casimirs are given by

$$C_{2SU_{sf}(n)} = \frac{1}{2} \left[\sum_{i=1}^n f_i(f_i + n + 1 - 2i) - \frac{1}{n} \left(\sum_{i=1}^n f_i \right)^2 \right]. \quad (16)$$

In Table 9, we give the expectation values of the Casimir operators $C_{2SU_{sf}(6)}$ and $C_{2SU_f(3)}$ for the allowed pentaquark configurations.

The last two terms in Eq. (15) correspond to the Gell-Mann-Okubo mass formula that describes the splitting within a flavour multiplet [37]. This formula was extended by Gürsey and Radicati [38] to include the terms proportional to B and C that depend on the spin and the flavour representations, which in turn was generalized further to include the spin-flavour term proportional to A as well [35].

In many studies of multiquark configurations, effective spin-flavour hyperfine interactions have been used in CQM which schematically represents the Goldstone Boson Exchange (GBE) interaction between constituent quarks [18, 19, 22, 23]. An analysis of the strange and non-strange qqq baryon resonances in the collective stringlike model [35] and the hypercentral CQM [40] also showed evidence for the need of such type of interaction terms. If one neglects their radial dependence, the matrix elements of these interactions depend on the Casimirs of the $SU_{sf}(6)$ spin-flavour, the $SU_f(3)$ flavour and the $SU_s(2)$ spin groups [19]

$$\left\langle \sum_{i < j}^n (\vec{\lambda}_i \cdot \vec{\lambda}_j)(\vec{\sigma}_i \cdot \vec{\sigma}_j) \right\rangle = 4C_{2SU_{sf}(6)} - 2C_{2SU_f(3)} - \frac{4}{3}s(s+1) - 8n. \quad (17)$$

where n is the number of quarks.

The energy splittings within a given multiplet induced by Eq. (17) have the same structure as the Gürsey-Radicati formula of Eq. (15), with the exception of the Gell-Mann-Okubo term. The constant with the number of quarks cancels out when evaluating energy differences. The dependence on the different quark numbers is taken into account by the fact that the eigenvalues of the Casimirs for the qqq or $qqqq\bar{q}$ states can be very different. The interaction of Eq. (17) is not the most general one. For instance, the presence of an explicit spin-spin interaction would modify the $-4/3$ coefficient.

In Eqs. (14) and (15) we have made a very strong approximation: we have neglected the spatial dependence of the $SU_{sf}(6)$ breaking part. As a consequence, there is no $SU_{sf}(6)$ mixing. The kind of problems that can arise neglecting the spatial dependence in the $SU_{sf}(6)$ breaking interaction is discussed by Jennings and Maltman [17] for two of the models in the literature, the Goldstone boson model and the bag model.

The average energy of $SU_{sf}(6)$ multiplets depends on the orbital part M_{orb} and on the term linear in the $SU_{sf}(6)$ Casimir, while the terms proportional to B , C , D and E give the splittings inside the multiplet. At the moment, there is experimental evidence for two pentaquark states. This is not sufficient to determine all parameters in the mass formula, and then to predict the masses of other pentaquarks. For this reason we use the values of the parameters determined from the three-quark spectrum, assuming that the coefficients in the GR are the same for different quark systems. Clearly, new experimental data on the pentaquark states will allow to determine how different can be the parameters relevant for the pentaquark spectrum with respect to the qqq ones.

In the case of the qqq system, the coefficients B , C , D and E can be obtained from the mass differences of selected pairs of baryon resonances [40]

$$M_{\Delta(1232)} - M_{N(938)} = 3(B + C + E),$$

$$\begin{aligned}
M_{N(1650)} - M_{N(1535)} &= 3C , \\
4M_{N(938)} - M_{\Sigma(1193)} - 3M_{\Lambda(1116)} &= 4D , \\
M_{\Sigma(1193)} - M_{\Lambda(1116)} &= 2E ,
\end{aligned} \tag{18}$$

leading to the numerical values

$$\begin{aligned}
B &= 21.2 \text{ MeV} , \\
C &= 38.3 \text{ MeV} , \\
D &= -197.3 \text{ MeV} , \\
E &= 38.5 \text{ MeV} .
\end{aligned} \tag{19}$$

The coefficients we have so obtained can be used for a preliminary evaluation of the splittings within any $SU_{\text{sf}}(6)$ multiplet, assuming that they do not depend on the quark system, just as is the case for the hyperfine interaction of Eq. (17). The eigenvalues of the Casimirs for the qqq or $qqqq\bar{q}$ systems are different (see Table 9) and in this way the presence of a different quark structure is taken into account.

We use the Gürsey-Radicati formula for the calculation of the energy splittings of the exotic pentaquark states, using the constant M_0 in order to normalize the energy scale to the observed mass of the Θ^+ . The results are shown in Table 10, where neither the M_{orb} nor the $A C_{2SU_{\text{sf}}(6)}$ terms have been introduced. Table 10 shows that for all spin-flavour configurations the lowest pentaquark state is characterized by $^2\bar{10}$, i.e. a flavour anti-decuplet [33] state with spin $s = 1/2$ and isospin $I = 0$, in agreement with the available experimental data which indicate that the $\Theta^+(1540)$ is an isosinglet [3]. For all spin-flavour configurations, there are other lowlying excited pentaquark states belonging to the 27-plet at 1660 MeV and 1775 MeV. The anti-decuplet state with strangeness $S = -2$ ($Y = -1$) and isospin $I = 3/2$ is calculated at an energy of 2305 MeV, to be compared with the recently observed resonance at 1862 MeV [8] which was suggested as a candidate for the $\Xi_{3/2}$ exotic with charge $Q = -2$.

Another important consequence of the use of a ‘diagonal’ form of the interactions in Eq. (15) is that the structure of the wave functions does not depend on the values of the coefficients. A change in the coefficients causes a shift in the energies, but does not modify the wave functions.

The degeneracy of the multiplets in Table 10 can be eliminated if one considers the contributions from the Casimir of $SU(6)$ and from the space term M_{orb} . For the consistent treatment of the latter one needs a specific model, but this is beyond the scope of this work. Nevertheless, we shall present some general arguments in the next section which are relevant for the spin and parity of the ground state pentaquark. Here we concentrate ourselves on the effects of the term linear in A in Eq. (15) on the energy splittings of pentaquark states. The value of the coefficient A can be determined, analogously to what has been done in connection with Eq. (19), from the energy difference between the lowest S_{11} resonance and the Roper

$$M_{N(1535)} - M_{N(1440)} = 3A + \Delta M_{\text{orb}} . \tag{20}$$

ΔM_{orb} is the orbital contribution to the mass difference, and can be taken from the $SU(6)$ invariant energies provided by the HCQM [35, 40], which leads to a value of $A = 55.1$ MeV. The positive sign of A is in agreement with the sign used in previous studies of baryons as qqq configurations [35].

In Table 11 we present the spin-flavour contribution to the energies of all exotic pentaquark states for the four allowed $SU_{\text{sf}}(6)$ spin-flavour multiplets. The effect of the spin-flavour term shifts the different $SU_{\text{sf}}(6)$ multiplets with respect to one another, without changing their internal structure. The lowest pentaquark state has the labels $^2\bar{10}$, i.e. is an anti-decuplet state with spin $s = 1/2$ and isospin $I = 0$, belonging to the $[51111]_{700}$ multiplet. The parity of this state is positive.

In the next section, we discuss the effect of orbital excitation energies on the angular momentum and parity of the ground state pentaquark. It is important to note that, irrespective of the orbital contribution to the mass, the ground state pentaquark is an anti-decuplet flavour state with spin $s = 1/2$ and isospin $I = 0$.

3.1 Spin and parity of the ground state pentaquark

What are the consequences of these calculations for the spin and parity of the $\Theta^+(1540)$? This depends in part on the assignment of quantum numbers, and in part on the choice of a particular model to describe the orbital motion. In the following we identify the $\Theta^+(1540)$ resonance with the ground state exotic pentaquark configuration.

The treatment of the orbital part is very much dependent on the choice of a specific dynamical model (harmonic oscillator, Skyrme, soliton, stringlike, hypercentral, ...). We consider a simple model in which the orbital motion of the pentaquark is limited to excitations up to $N = 1$ quantum. The model space consists of five states: an S -wave state with $L^P = 0^+$ and A_1 symmetry for the four quarks, and four excited P -wave states with $L^P = 1^-$, three of which correspond to excitations in the relative coordinates of the four-quark subsystem and the fourth to an excitation in the relative coordinate between the four-quark subsystem and the antiquark. As a consequence of the S_4 permutation symmetry of the four quarks, the first three excitations form a degenerate triplet with three-fold F_2 symmetry, and the fourth has A_1 symmetry. In summary, the states in this simple model for the orbital motion are characterized by angular momentum L , parity p and \mathcal{T}_d symmetry t : $L_t^P = 0_{A_1}^+, 1_{F_2}^-$ and $1_{A_1}^-$. The total angular momentum of the pentaquark state is given by $\vec{J} = \vec{L} + \vec{s}$, whereas the parity is opposite to that of the orbital excitation due to the negative intrinsic parity of the $q^4\bar{q}$ configuration. According to Table 8, the exotic spin-flavour states associated with the orbital states $L_t^P = 0_{A_1}^+$ and $1_{A_1}^-$ belong to the $[42111]_{1134}$ representation, whereas the state $L_t^P = 1_{F_2}^-$ gives rise to exotic pentaquark states belonging to the $[51111]_{700}$, $[42111]_{1134}$, $[33111]_{560}$ and $[32211]_{540}$ configurations. In Fig. 4 we show a schematic spectrum of the orbital excitations of the pentaquark up to $N = 1$ quantum, which depends on the excitation energies, Δ_1 and Δ_2

$$\begin{aligned}\Delta_1 &= E_{\text{orb}}(1_{F_2}^-) - E_{\text{orb}}(0_{A_1}^+) , \\ \Delta_2 &= E_{\text{orb}}(1_{A_1}^-) - E_{\text{orb}}(0_{A_1}^+) .\end{aligned}\tag{21}$$

The energy of a given spin-flavour multiplet depends on the orbital excitation energies Δ_1 and Δ_2 , and the coefficient A , while the terms proportional to B , C , D and E give the splitting inside the multiplet. The quantum numbers of the ground state depend on the relative size of Δ_1 and A . Its parity is opposite to that of the orbital excitation due to the negative intrinsic parity of $q^4\bar{q}$ configurations.

For $\Delta_1 > 4A = 220$ MeV, the ground state pentaquark is associated with the orbital state with $L_t^P = 0_{A_1}^+$ and the $^2\overline{10}$ anti-decuplet state of the $[42111]$ multiplet. In this case, the angular momentum and parity of the ground state pentaquark are $J^P = 1/2^-$. Another possible identification of the observed Θ^+ is provided by the $[42111]$ anti-decuplet state with $s = 3/2$, in which case the ground state would have $J^P = 3/2^-$. This would imply that, because of the positive value of the spin splitting coefficient C in Eqs. (15) and (19), there should be a another pentaquark state with $s = 1/2$ and $J^P = 1/2^-$ at an energy lower than the one observed. At the moment, there is no experimental evidence for such an exotic state for which reason this identification seems to be ruled out.

For $\Delta_1 < 4A = 220$ MeV, the parity of the lowest pentaquark state would be positive, since the ground state now corresponds to the orbital excitation with $L_t^P = 1_{F_2}^-$ and the $^2\overline{10}$ flavour anti-decuplet of the $[51111]$ multiplet. In the absence of spin-orbit splitting, we find in this case a ground state doublet with angular momentum and parity $J^P = 1/2^+, 3/2^+$. The calculation of Table 11 belongs to this class since $\Delta_1 = 0$.

4 Summary, conclusions and outlook

In this work, we have constructed a classification scheme of the pentaquark states in terms of $SU_{\text{sf}}(6)$ spin-flavour multiplets, and their flavour and spin content in terms of $SU_{\text{f}}(3)$ and $SU_{\text{s}}(2)$ states. Exotic pentaquark states can be found only in the flavour anti-decuplets, 27-plets and 35-plets. Moreover, we

have discussed the permutation symmetry properties of both the spin-flavour and orbital parts of the $qqqq$ subsystem. In order to obtain the total wave function, the spin-flavour part has been combined with the colour and orbital contributions in such a way that the total pentaquark wave function is a colour singlet and is antisymmetric under the interchange of any of the four quarks. This classification scheme is general and complete, and may be helpful for both experimental, CQM and lattice QCD studies. In particular, the constructed basis for pentaquark states will enable to solve the eigenvalue problem for a definite dynamical model. This is valid not only for Constituent Quark Models, but also for diquark-diquark-antiquark approaches, for which the basis is a subset of the one we have constructed.

As an application we have calculated the mass spectrum of exotic pentaquark states with the Gürsey-Radicati mass formula which corresponds to the dynamical symmetry described by the chain of subgroups

$$SU_{sf}(6) \supset SU_f(3) \otimes SU_s(2) \supset SU_I(2) \otimes U_Y(1) \otimes SU_s(2) , \quad (22)$$

and encodes the slightly broken symmetries of the strong interactions. In the assumption of a GR formula we have neglected the radial dependence of the $SU_{sf}(6)$ spin-flavour quark interaction. The problems that arise from this kind of approximation have been discussed in the literature, nevertheless similar methods have been used in other studies of pentaquark states. In principle, the coefficients of the GR applied to the $qqqq\bar{q}$ system should be obtained from a fit of the pentaquark spectrum. This is however not possible at the moment, since we know at most two pentaquark states. Therefore, under the assumption that the coefficients do not depend strongly on the structure of the quark system, we have calculated the pentaquark spectrum using the coefficients taken from a prior study of qqq baryons [40], in order to get an idea of the general features of the spectrum. As a result we find that the lowest pentaquark is always an $2\bar{1}0$ anti-decuplet state with isospin $I = 0$, in agreement with experimental evidence that the $\Theta^+(1540)$ is an isosinglet. We also presented some preliminary results based on a generalized Gürsey-Radicati mass formula which includes the invariant of the $SU_{sf}(6)$ spin-flavour group, and a simple schematic model for the orbital excitations up to $N = 1$ quantum.

The angular momentum and parity of the ground state exotic pentaquark depends on the relative contribution of the orbital and spin-flavour parts of the mass operator. We find that if the splitting due to the $SU_{sf}(6)$ spin-flavour term is large compared to that between the orbital states, the ground state pentaquark has positive parity [18, 19, 23], whereas for a relatively small spin-flavour splitting the parity of the lowest pentaquark state becomes negative. We notice that, in case of a positive parity ground state, there is a doublet with $J^P = 1/2^+, 3/2^+$ which, in the presence of a spin-orbit coupling term, would give rise to a pair of peaks. The effect of specific dynamical models on the pentaquark spectrum in general, and on the properties of its ground state in particular, using a space dependent $SU(6)$ breaking interaction, will be presented in more detail in a separate publication [32].

The spectroscopy of exotic baryons will be a key testing ground for models of baryons and their structure. Especially the determination of the angular momentum and parity of the $\Theta^+(1540)$ will allow to distinguish between different approaches [17]. Most theoretical studies predict a positive parity for the Θ^+ [14, 15, 18, 19, 21, 23, 26], but there is also evidence for a negative parity from recent work on QCD sum rules [27] and lattice QCD [29]. Other challenges include the search for excited exotics.

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Table 1: Symmetry properties of three-quark states




D_3	\sim	S_3	Young tableau	Multiplicity	Dimension		
					$SU(6)$	$SU(3)$	$SU(2)$
A_1	\sim	$[3]$		1	56	10	4
E	\sim	$[21]$		2	70	8	2
A_2	\sim	$[111]$		1	20	1	—

 Table 2: Spin-flavour classification of q^3 states

D_3	$SU_{\text{sf}}(6)$	\supset	$SU_{\text{f}}(3)$	\otimes	$SU_{\text{s}}(2)$
A_1	$[3]_{56}$		$[3]_{10}$	\otimes	$[3]_4$
			$[21]_8$	\otimes	$[21]_2$
E	$[21]_{70}$		$[3]_{10}$	\otimes	$[21]_2$
			$[21]_8$	\otimes	$[3]_4$
			$[21]_8$	\otimes	$[21]_2$
			$[111]_1$	\otimes	$[21]_2$
A_2	$[111]_{20}$		$[21]_8$	\otimes	$[21]_2$
			$[111]_1$	\otimes	$[3]_4$

Table 3: Symmetry properties of four-quark $SU(6)$ states

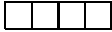




\mathcal{T}_d	\sim	S_4	Young tableau	Multiplicity	Dimension		
					$SU(6)$	$SU(3)$	$SU(2)$
A_1	\sim	$[4]$		1	126	15	5
F_2	\sim	$[31]$		3	210	15	3
E	\sim	$[22]$		2	105	6	1
F_1	\sim	$[211]$		3	105	3	—
A_2	\sim	$[1111]$		1	15	—	—

Table 4: Spin-flavour decomposition of q^4 states

D_3	\mathcal{T}_d	$SU_{\text{sf}}(6)$	\supset	$SU_{\text{f}}(3)$	\otimes	$SU_{\text{s}}(2)$
A_1	A_1	$[4]_{126}$		$[4]_{15}$ $[31]_{15}$ $[22]_6$	\otimes \otimes \otimes	$[4]_5$ $[31]_3$ $[22]_1$
$A_1 + E$	F_2	$[31]_{210}$		$[4]_{15}$ $[31]_{15}$ $[31]_{15}$ $[31]_{15}$ $[22]_6$ $[211]_3$ $[211]_3$	\otimes \otimes \otimes \otimes \otimes \otimes \otimes	$[31]_3$ $[4]_5$ $[31]_3$ $[22]_1$ $[31]_3$ $[22]_1$ $[31]_3$
E	E	$[22]_{105}$		$[4]_{15}$ $[31]_{15}$ $[22]_6$ $[22]_6$ $[211]_3$	\otimes \otimes \otimes \otimes \otimes	$[22]_1$ $[31]_3$ $[4]_5$ $[22]_1$ $[31]_3$
$E + A_2$	F_1	$[211]_{105}$		$[31]_{15}$ $[31]_{15}$ $[22]_6$ $[211]_3$ $[211]_3$ $[211]_3$	\otimes \otimes \otimes \otimes \otimes \otimes	$[31]_3$ $[22]_1$ $[31]_3$ $[4]_5$ $[31]_3$ $[22]_1$
A_2	A_2	$[1111]_{15}$		$[22]_6$ $[211]_3$	\otimes \otimes	$[22]_1$ $[31]_3$

Table 5: Allowed spin, flavour and spin-flavour pentaquark states

	$qqqq\bar{q}$	Dimension	$S_4 \sim \mathcal{T}_d$
spin	[5]	6	A_1
	[41]	4	A_1, F_2
	[32]	2	F_2, E
flavour	[51]	35-plet	A_1
	[42]	27-plet	F_2
	[33]	antidecuplet	E
	[411]	decuplet	A_1, F_2
	[321]	octet	F_2, E, F_1
	[222]	singlet	F_1
spin-flavour	[51111]	700	A_1
	[411111]	56	A_1, F_2
	[42111]	1134	F_2
	[321111]	70	F_2, E, F_1
	[33111]	560	E
	[32211]	540	F_1
	[222111]	20	F_1, A_2
	[22221]	70	A_2

Table 6: Continued

\mathcal{T}_d	$SU_{\text{sf}}(6)$	\supset	$SU_{\text{f}}(3)$	\otimes	$SU_{\text{s}}(2)$
E	$[33111]_{560}$		$[51]_{35}$ $[42]_{27}$ $[42]_{27}$ $[33]_{10}$ $[33]_{10}$ $[33]_{10}$ $[411]_{10}$ $[411]_{10}$ $[321]_8$ $2([321]_8)$ $2([321]_8)$ $[222]_1$	\otimes \otimes \otimes \otimes \otimes \otimes \otimes \otimes \otimes \otimes \otimes \otimes	$[32]_2$ $[41]_4$ $[32]_2$ $[5]_6$ $[41]_4$ $[32]_2$ $[41]_4$ $[32]_2$ $[5]_6$ $[41]_4$ $[32]_2$ $[41]_4$
F_1	$[32211]_{540}$		$[42]_{27}$ $2([42]_{27})$ $[33]_{10}$ $[33]_{10}$ $[411]_{10}$ $[411]_{10}$ $[321]_8$ $3([321]_8)$ $3([321]_8)$ $[222]_1$ $[222]_1$ $[222]_1$	\otimes \otimes \otimes \otimes \otimes \otimes \otimes \otimes \otimes \otimes \otimes \otimes	$[41]_4$ $[32]_2$ $[41]_4$ $[32]_2$ $[41]_4$ $[32]_2$ $[5]_6$ $[41]_4$ $[32]_2$ $[5]_6$ $[41]_4$ $[32]_2$
$F_1 + A_2$	$[222111]_{20}$		$[321]_8$ $[222]_1$	\otimes \otimes	$[32]_2$ $[41]_4$
A_2	$[22221]_{70}$		$[33]_{10}$ $[321]_8$ $[321]_8$ $[222]_1$	\otimes \otimes \otimes \otimes	$[32]_2$ $[41]_4$ $[32]_2$ $[32]_2$

Table 7: $q^4\bar{q}$ pentaquark states with exotic quantum numbers. The electric charge is $Q = I_3 + Y/2$. Notation as in [31].

$SU_f(3)$	Y	I	Q	Flavour States	Notation
$[33]_{10}$	2	0	1	$dduu\bar{s}$	Θ
	-1	3/2	-2,1	$ddss\bar{u}, uuss\bar{d}$	$\Xi_{3/2}$
$[42]_{27}$	2	1	0,1,2	$dddu\bar{s}, dduu\bar{s}, duuu\bar{s}$	Θ_1
	0	2	-2,2	$ddds\bar{u}, uuus\bar{d}$	Σ_2
	-1	3/2	-2,1	$ddss\bar{u}, uuss\bar{d}$	$\Xi_{3/2}$
	-2	1	-2,0	$dsss\bar{u}, usss\bar{d}$	Ω_1
$[51]_{35}$	2	2	-1,0,1,2,3	$dddd\bar{s}, dddu\bar{s}, dduu\bar{s}, duuu\bar{s}, uuus\bar{d}$	Θ_2
	1	5/2	-2, 3	$dddd\bar{u}, uuus\bar{d}$	$\Delta_{5/2}$
	0	2	-2, 2	$ddds\bar{u}, uuus\bar{d}$	Σ_2
	-1	3/2	-2, 1	$ddss\bar{u}, uuss\bar{d}$	$\Xi_{3/2}$
	-2	1	-2, 0	$dsss\bar{u}, usss\bar{d}$	Ω_1
	-3	1/2	-2,-1	$ssss\bar{u}, sssd\bar{d}$	Φ

Table 8: Decomposition of the orbital-spin-flavour wave function with F_2 symmetry into orbital and spin-flavour parts. In the last column the pentaquark configurations that contain exotic states are shown.

Orbital Symmetry	Spin-Flavour Symmetry	$q^4\bar{q}$ Configuration with Exotic States
A_1	F_2	[42111]
F_2	A_1	[51111]
	F_2	[42111]
	E	[33111]
	F_1	[32211]
E	F_2	[42111]
	F_1	[32211]
F_1	A_2	[22221]
	F_2	[42111]
	E	[33111]
	F_1	[32211]
A_2	F_1	[32211]

Table 9: Eigenvalues of the $C_{2SU_{sf}(6)}$ and $C_{2SU_f(3)}$ Casimir operators

spin-flavour	$C_{2SU_{sf}(6)}$	flavour	$C_{2SU_f(3)}$
[51111] ₇₀₀	81/4	[51] ₃₅	12
[411111] ₅₆	45/4	[42] ₂₇	8
[42111] ₁₁₃₄	65/4	[33] ₁₀	6
[321111] ₇₀	33/4	[411] ₁₀	6
[33111] ₅₆₀	57/4	[321] ₈	3
[32211] ₅₄₀	49/4	[222] ₁	0
[222111] ₂₀	21/4		
[22221] ₇₀	33/4		

Table 10: Mass splittings of exotic pentaquark states within a $SU_{\text{sf}}(6)$ multiplet calculated using Eq. (15) with the parameters of Eq. (19). The pentaquark ground state is normalized to the observed mass of the $\Theta^+(1540)$ resonance. The orbital excitations are taken to be degenerate. The states are labeled by their spin s , hypercharge Y , isospin I , spin-flavour multiplet $[f]$ and orbital excitation L_t^p . The notation is the same as in Table 7.

$SU_f(3)$	s	Y	I	Notation	Mass (MeV)			
					$[51111]$ $1_{F_2}^-$	$[42111]$ $0_{A_1}^+, 1_{A_1, F_2}^-$	$[33111]$ $1_{F_2}^-$	$[32211]$ $1_{F_2}^-$
$[33]_{10}$	$1/2$	2	0	Θ	1540	1540	1540	1540
		-1	$3/2$	$\Xi_{3/2}$	2305	2305	2305	2305
$[33]_{10}$	$3/2$	2	0	Θ		1655	1655	1655
		-1	$3/2$	$\Xi_{3/2}$		2420	2420	2420
$[33]_{10}$	$5/2$	2	0	Θ			1846	
		-1	$3/2$	$\Xi_{3/2}$			2612	
$[42]_{27}$	$1/2$	2	1	Θ_1	1659	1659	1659	1659
		0	2	Σ_2	2247	2247	2247	2247
		-1	$3/2$	$\Xi_{3/2}$	2348	2348	2348	2348
		-2	1	Ω_1	2449	2449	2449	2449
$[42]_{27}$	$3/2$	2	1	Θ_1	1774	1774	1774	1774
		0	2	Σ_2	2361	2361	2361	2361
		-1	$3/2$	$\Xi_{3/2}$	2461	2461	2461	2461
		-2	1	Ω_1	2564	2564	2564	2564
$[42]_{27}$	$5/2$	2	1	Θ_1		1966		
		0	2	Σ_2		2553		
		-1	$3/2$	$\Xi_{3/2}$		2654		
		-2	1	Ω_1		2755		

Table 10: Continued

$SU_f(3)$	s	Y	I	Notation	Mass (MeV)			
					$[51111]$ $1_{F_2}^-$	$[42111]$ $0_{A_1}^+, 1_{A_1, F_2}^-$	$[33111]$ $1_{F_2}^-$	$[32211]$ $1_{F_2}^-$
$[51]_{35}$	$1/2$	2	2	Θ_2		1898	1898	
			1	$\Delta_{5/2}$		2230	2230	
			0	Σ_2		2331	2331	
			-1	$\Xi_{3/2}$		2432	2432	
			-2	Ω_1		2533	2533	
			-3	Φ		2634	2634	
$[51]_{35}$	$3/2$	2	2	Θ_2	2013	2013		
			1	$\Delta_{5/2}$	2345	2345		
			0	Σ_2	2446	2446		
			-1	$\Xi_{3/2}$	2547	2547		
			-2	Ω_1	2648	2648		
			-3	Φ	2749	2749		
$[51]_{35}$	$5/2$	2	2	Θ_2	2205			
			1	$\Delta_{5/2}$	2537			
			0	Σ_2	2638			
			-1	$\Xi_{3/2}$	2739			
			-2	Ω_1	2840			
			-3	Φ	2941			

Table 11: Spin-flavour contribution to the masses of exotic pentaquark states calculated using Eq. (15) with the parameters of Eq. (19) and $A = 55.1$ MeV. The pentaquark ground state is normalized to the observed mass of the $\Theta^+(1540)$ resonance. The notation and the labeling of the states is the same as in Table 10. The orbital excitations are taken to be degenerate.

$SU_f(3)$	s	Y	I	Notation	Mass (MeV)			
					$[51111]$ $1_{F_2}^-$	$[42111]$ $0_{A_1}^+, 1_{A_1, F_2}^-$	$[33111]$ $1_{F_2}^-$	$[32211]$ $1_{F_2}^-$
$[33]_{10}$	$1/2$	2	0	Θ	1320	1540	1650	1760
		-1	$3/2$	$\Xi_{3/2}$	2085	2305	2415	2526
$[33]_{10}$	$3/2$	2	0	Θ		1655	1765	1875
		-1	$3/2$	$\Xi_{3/2}$		2420	2530	2640
$[33]_{10}$	$5/2$	2	0	Θ			1957	
		-1	$3/2$	$\Xi_{3/2}$			2722	
$[42]_{27}$	$1/2$	2	1	Θ_1	1439	1659	1770	1880
		0	2	Σ_2	2026	2247	2357	2467
		-1	$3/2$	$\Xi_{3/2}$	2127	2348	2458	2568
		-2	1	Ω_1	2228	2449	2559	2669
$[42]_{27}$	$3/2$	2	1	Θ_1	1554	1774	1885	1995
		0	2	Σ_2	2141	2361	2472	2582
		-1	$3/2$	$\Xi_{3/2}$	2242	2462	2573	2683
		-2	1	Ω_1	2343	2564	2674	2784
$[42]_{27}$	$5/2$	2	1	Θ_1		1966		
		0	2	Σ_2		2553		
		-1	$3/2$	$\Xi_{3/2}$		2654		
		-2	1	Ω_1		2755		

Table 11: Continued

$SU_f(3)$	s	Y	I	Notation	Mass (MeV)			
					$[51111]$ $1_{F_2}^-$	$[42111]$ $0_{A_1}^+, 1_{A_1, F_2}^-$	$[33111]$ $1_{F_2}^-$	$[32211]$ $1_{F_2}^-$
$[51]_{35}$	$1/2$	2	2	Θ_2		1898	2008	
			1	$\Delta_{5/2}$		2230	2340	
			0	Σ_2		2331	2442	
			-1	$\Xi_{3/2}$		2432	2543	
			-2	Ω_1		2533	2644	
			-3	Φ		2634	2745	
$[51]_{35}$	$3/2$	2	2	Θ_2	1793	2013		
			1	$\Delta_{5/2}$	2125	2345		
			0	Σ_2	2226	2446		
			-1	$\Xi_{3/2}$	2327	2547		
			-2	Ω_1	2428	2648		
			-3	Φ	2529	2749		
$[51]_{35}$	$5/2$	2	2	Θ_2	1984			
			1	$\Delta_{5/2}$	2316			
			0	Σ_2	2417			
			-1	$\Xi_{3/2}$	2518			
			-2	Ω_1	2619			
			-3	Φ	2720			

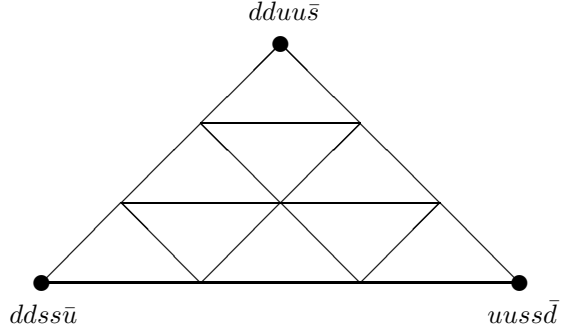


Figure 1: $SU(3)$ flavour multiplet $[33]_{10}$ with E symmetry. The isospin-hypercharge multiplets are $(I, Y) = (0, 2), (1/2, 1), (1, 0)$ and $(3/2, -1)$. Exotic states are indicated with \bullet .

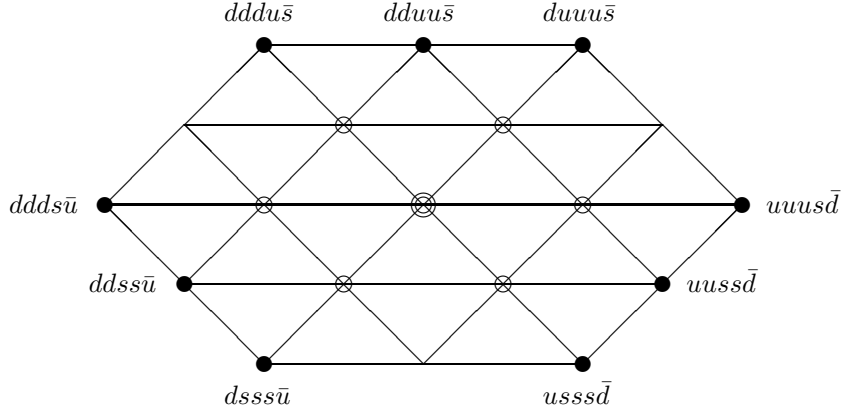


Figure 2: $SU(3)$ flavour multiplet $[42]_{27}$ with F_2 symmetry. The isospin-hypercharge multiplets are $(I, Y) = (1, 2), (3/2, 1), (1/2, 1), (2, 0), (1, 0), (0, 0), (3/2, -1), (1/2, -1)$ and $(1, -2)$. Exotic states are indicated with \bullet .

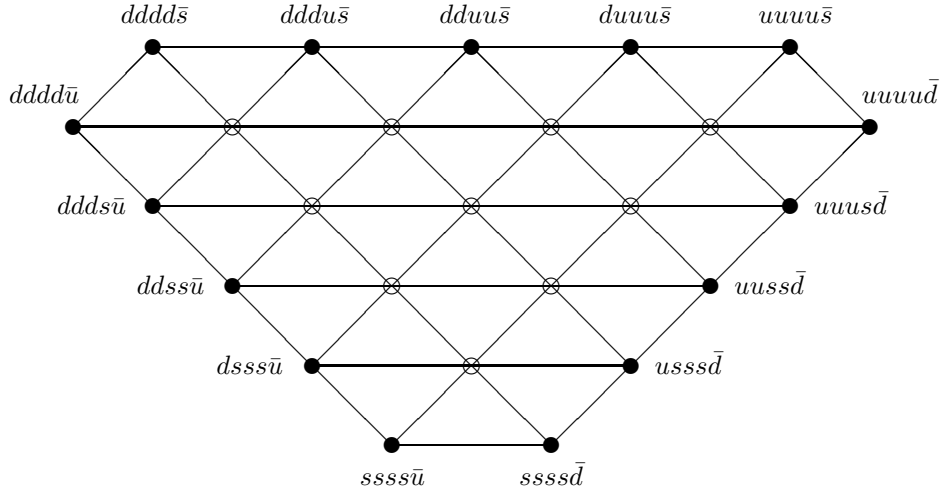


Figure 3: $SU(3)$ flavour multiplet $[51]_{35}$ with A_1 symmetry. The isospin-hypercharge multiplets are $(I, Y) = (2, 2), (5/2, 1), (3/2, 1), (2, 0), (1, 0), (3/2, -1), (1/2, -1), (1, -2), (0, -2)$ and $(1/2, -3)$. Exotic states are indicated with \bullet .

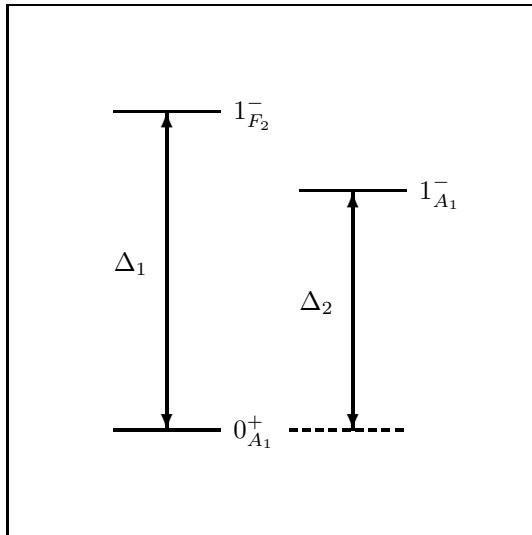


Figure 4: Orbital excitations of the pentaquark up to $N = 1$ quantum. The states are labeled by angular momentum, parity and \mathcal{T}_d symmetry L_t^p .